hand side of the inequality, (1) can be rearranged to yield the form

$$(1-|S_{21}|^2)(1-|S_{12}|^2)-|S_{11}|^2 \geq |S_{22}|^2(1-|S_{11}|^2) + 2|S_{22}||S_{11}||S_{12}||S_{21}|\cos\theta \quad (2)$$

where

$$\theta = \theta_{11}+\theta_{22}-\theta_{21}-\theta_{12}. \quad (3)$$

Dividing both sides of (2) by the factor $(1-|S_{11}|^2)$ and then completing the square on the right-hand side gives the following equation:

$$\frac{(1-|S_{21}|^2)(1-|S_{12}|^2)-|S_{11}|^2}{1-|S_{11}|^2}+\frac{|S_{11}|^2|S_{21}|^2|S_{12}|^2\cos^2\theta}{(1-|S_{11}|^2)^2} \geq \left(|S_{22}|+\frac{|S_{11}||S_{21}||S_{12}|\cos\theta}{(1-|S_{11}|^2)}\right)^2. \quad (4)$$

After taking the positive square root of (4) and rearranging terms we obtain

$$\sqrt{\frac{(1-|S_{21}|^2)(1-|S_{12}|^2)-|S_{11}|^2}{1-|S_{11}|^2}+\frac{|S_{11}|^2|S_{21}|^2|S_{12}|^2\cos^2\theta}{(1-|S_{11}|^2)^2}} - \frac{|S_{11}||S_{21}||S_{12}|\cos\theta}{1-|S_{11}|^2} \geq |S_{22}|. \quad (5)$$

The upper bound for (5) occurs for $\theta=\pi$ and is

$$\frac{\sqrt{(1-|S_{21}|^2-|S_{11}|^2)(1-|S_{12}|^2-|S_{11}|^2)}+|S_{11}||S_{21}||S_{12}|}{1-|S_{11}|^2} \geq |S_{22}|. \quad (6)$$

Equation (6) differs from eq. (9) of Hindin by the sign in front of the square-root term.

Equation (9) in Hindin leads to a contradiction as can be seen by setting $|S_{11}|=0$. In this case it gives the result

$$|S_{22}| \leq -\sqrt{(1-|S_{12}|^2)(1-|S_{21}|^2)} \quad (7)$$

which is contradictory. For $|S_{11}|=0$, (6) of this letter gives the result

$$|S_{22}| \leq \sqrt{(1-|S_{12}|^2)(1-|S_{21}|^2)} \quad (8)$$

which is not contradictory.

For a reciprocal network, (6) of this letter reduces to the form

$$\frac{|(1-|S_{12}|^2)-|S_{11}|^2|+|S_{11}||S_{12}|^2}{1-|S_{11}|^2} \geq |S_{22}|. \quad (9)$$

Using the inequality $1-|S_{12}|^2-|S_{11}|^2 \geq 0$ given in (1), (9) of this letter reduces to the form

$$1-\frac{|S_{12}|^2}{1+|S_{11}|} \geq |S_{22}| \quad (10)$$

which is Uhlir's eq. (12).

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MIC *Ku*-Band Upconverter

P. BURA

***Abstract*—An MIC *S*- to *Ku*-band upper-sideband upconverter has shown a pump efficiency of 25 percent and 3.8-dB signal gain. When used as a lower-sideband upconverter, gains of 13 dB and 100-MHz bandwidth were measured.**

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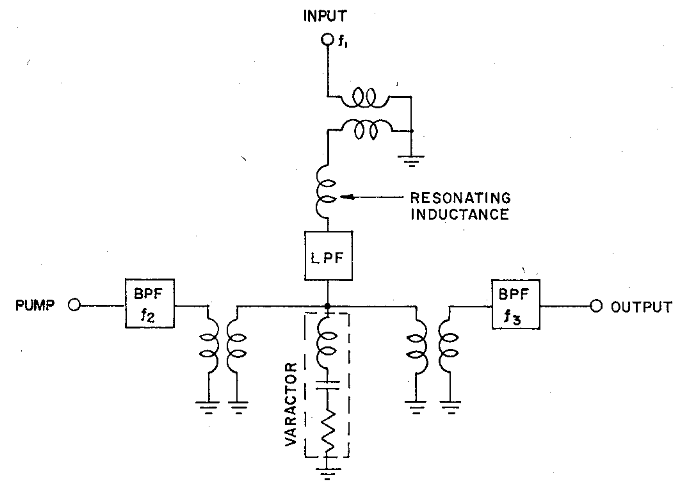

Fig. 1. Equivalent circuit of the upconverter.

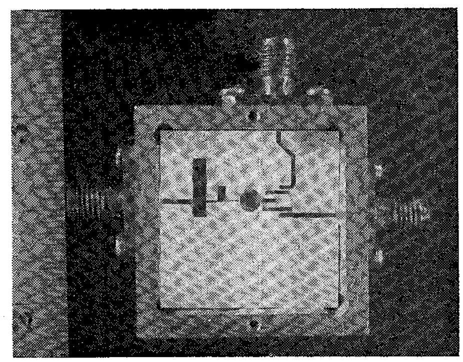

Fig. 2. MIC upconverter.

The development of the MIC *Ku*-band upconverter represents frequency extension of the previously reported *S*-band circuit [1]. The equivalent circuit of the unbalanced upconverter is shown in Fig. 1. It consists of the varactor in parallel with the input, output, and pump frequency circuits. Each circuit transforms the 50-Ω line impedance to the value required for the optimum upconverter performance and, at the same time, presents an open circuit at the other two frequencies.

The MIC upconverter, using Cr–Au metallization on a 25-mil alumina substrate, is shown in Fig. 2. The varactor was chosen to be self-resonant between the pump and the output frequencies, so that no additional tuning was required at those frequencies. Asymmetrical coupled transmission-line filters were used both for filtering and impedance transformation from $R_0=50$ Ω. The transformation is given by

$$R=\left(\frac{Z_{e2}-Z_{02}}{Z_{e1}-Z_{01}}\right)^2 R_0.$$

The values of the odd and even impedances determine the strip widths and spacings in the filters.

The signal input circuit has to resonate the varactor and to provide the correct input loading. This is achieved by means of a capacitive stub and a transforming section of a high impedance line. In addition, the input circuit contains a wide-band choke section and a pump reject stub. The upconverter design is based on the analysis of Penfield [2] and Grayzel [3].

To assure strictly reactive behavior, a constraint on the junction voltage is imposed, limiting the swing between the forward conduction and reverse breakdown. This is expressed by:

$$m_1+m_2+m_3 \leq 0.25$$

where m_1, m_2, and m_3 are elastance modulation ratios at component frequencies. This is a severely limiting restriction, as it assumes that elastance components reach their peak at the same instant.

Nelson [4] has derived a more realistic constraint based on 90° phase difference between the output charge and signal and pump charge, with the worst case phase condition between the latter two. It is given by

$$m_1 \cos\theta + \sqrt{m_2^2 + m_3^2 + 2m_2m_3 \sin\theta} \leq 0.25 \quad (1)$$

where

$$m_1 \sin\theta = \frac{m_2 m_3 \cos\theta}{\sqrt{m_2^2 + m_3^2 + 2m_2m_3 \sin\theta}}. \quad (2)$$

The two equations are not sufficient to determine the elastance modulation ratios. They can, however, be determined if we introduce additional relations, based on Manley–Rowe equations, between them and the component frequencies, viz.,

$$m_1^2 f_1 = m_2^2 f_2 = m_3^3 f_3. \quad (3)$$

Eliminating the modulation ratios, we are left with a cubic equation for $y = \sin\theta$

$$y^3 + \sqrt{\frac{f_3}{f_2}}\, y^2 - \frac{1}{2}\frac{f_1}{\sqrt{f_2 f_3}} = 0 \quad (4)$$

which can be solved for given frequencies. Equation (1) can be rewritten as

$$m_1 = \frac{0.25}{\cos\theta + \sqrt{\frac{f_1}{f_2} + \frac{f_1}{f_3} + \frac{2f_1}{\sqrt{f_2 f_3}}\sin\theta}} \quad (5)$$

and m_1 can be calculated for the known value of θ. The remaining modulation ratios are then obtained from (3).

With the knowledge of modulation ratios, the input and output loading impedance can be determined:

$$R_{\text{in}} = R_s\left(1 + \frac{m_1 f_c}{\sqrt{f_2 f_3}}\right) \quad (6)$$

$$R_{\text{out}} = R_s\left(\frac{m_1 f_c}{\sqrt{f_2 f_3}} - 1\right) \quad (7)$$

where

R_s varactor series resistance,
f_c varactor cutoff frequency (at breakdown).

In this case f_c equals 600 GHz at −10 V and R_s equals 2.32 Ω. The frequencies were

$$f_1 = 2.4 \text{ GHz} \qquad f_2 = 12.7 \text{ GHz} \qquad f_3 = 15.1 \text{ GHz}.$$

Equations (1)–(3) yielded

$$m_1 = 0.1536 \qquad m_2 = 0.0668 \qquad m_3 = 0.0612$$

with

$$R_{\text{in}} = 20.2\,\Omega \qquad R_{\text{out}} = 15.6\,\Omega.$$

The following results were obtained: $P_{\text{out}} = 10$ mW at 15.1 GHz for $P_{\text{in}} = 4.7$ mW at 2.4 GHz and $P_{\text{in}} = 40$ mW at 12.7 GHz, with the pump efficiency of

$$\eta_p = \frac{10}{40} = 25 \text{ percent}$$

and signal gain

$$G = \frac{10}{4.7} = 3.8 \text{ dB}.$$

The theoretical varactor efficiency is given by

$$\eta_v = \frac{f_3}{f_2}\,\frac{m_1 f_c - \sqrt{f_2 f_3}}{m_1 f_c + \sqrt{f_2 f_3}} = 0.878.$$

Thus the circuit efficiency is

$$\eta_c = \frac{0.25}{0.878} = 0.285 \quad \text{or 5.4-dB loss.}$$

The loss in the two filters amounts to 2.6 dB, thus leaving 2.8 dB as the loss in the remaining circuit.

Reversing the pump and output terminals, the same circuit can be used as a lower-sideband upconverter. This is a negative-resistance circuit and is capable of much higher signal gain. The transducer gain is given by

$$G = \frac{R_g R_L m^2 f_c^2}{4 f_1^2 R_s^2 \left[\left(\frac{R_g}{R_s} + 1\right)\left(\frac{R_L}{R_s} + 1\right) - \frac{m^2 f_c^2}{f_1 f_2}\right]}.$$

In this case $m = m_3 = 0.0612$ and $R_g = R_L = R_{\text{in}} = 20.2$ Ω. Substituting, we obtain $G = 2.5$ dB.

It is important to note that this corresponds to the saturation value of the gain, as the signal and idler modulation ratios are greater than those of the pump. By decreasing the former, we can increase the pump-modulation ratio and the upconverter gain. This, of course, is achieved by lowering the input signal level and increasing the pump level. Under these conditions, transucucer gains of 13 dB and 100 MHz or 4-percent 3-dB signal bandwidth were measured.

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Wide-Band Varactor-Tuned X-Band Gunn Oscillators in Full-Height Waveguide Cavity

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***Abstract*—Some results on X-band varactor-tuned Gunn oscillators in a full-height waveguide cavity are presented. It is shown that it is possible to obtain an electronic tuning range of over 1 GHz at X band with an appreciable output power level, which is also nearly constant with frequency. Results of FM noise measurements on one such oscillator are also reported.**

Wide-band varactor-tuned Gunn oscillators have previously been reported in the literature [1]–[3]. Lee and Hodgart [1] obtained 1-GHz electronic tuning in J band, while Smith and Crane [2] obtained 1.1-GHz electronic tuning in X band. Downing and Myers [3] achieved 1.95 GHz of electronic tuning range in X band with a reduced-height waveguide cavity. These authors used inherently low-Q cavities in the form of either reduced-height waveguide or coaxial line structures. Previous attempts [3] to achieve a wide varactor tuning range in a full-height waveguide cavity at X band resulted in a maximum tuning of 200 MHz. However, by appropriately positioning the Gunn and varactor devices, an electronic tuning range in excess of 1 GHz has been realized here.

The experiments were conducted with Mullard Type CXY 19 Gunn devices and silicon tuning varactors. Both are encapsulated devices in the standard S4 package. Typical Gunn-device parameters are: $V_T = 4.75$ V, $I_T = 600$ mA, $V_G = 12.0$ V, $I_G = 450$ mA, $P_0 = 150$ mW in a standard test cavity. The devices were mounted on cylindrical post structures in a standard WG 16 waveguide block with suitable biasing arrangements incorporated. A moveable short circuit was used to form the waveguide cavity, and no matching elements

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